The "Len-Tenna"
Physical optics and beam former design as converging disciplines

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ABSTRACT

We are witnessing an astonishing growth in technologies related to the expansion of beam former hardware to higher frequencies, larger apertures, and wider fractional bandwidths. At the same time we are seeing, as a result of MEMS and nanotechnology, the development of optical systems that are looking curiously like phased arrays. In this paper we explore the similarities between optics and antenna design as employed in radar systems. We will see that many principles of optics can be viewed using beam-forming concepts familiar to radar engineers. We will discuss Fresnel lenses, subspace, and near field beam formers.

1. *INTRODUCTION*

Beam-forming as applied in radar and comm.-unications, and physical optics as applied to optical telescopes, seem on the surface to have little in common other than the obvious fact they both involve electromagnetic waves. We are here going to show that this disparity is illusory and that significant insights, perhaps even new systems, can emerge from considerations of their commonality[1]. To begin consider a simple prism, as shown in figure 1, and contrasted with a phased array antenna. Note that the bending of light in the prism, usually referred to as Snell's law, is usually "blamed" on dielectric coefficients and continuity requirement of electric fields as mandated by Maxwell's equations [3]. This approach tends to not foster insight for radar signal processing engineers who are much more comfortable with steering vectors than PDEs. At the level of the diffraction limit, that is when we consider optical systems from a wave front perspective, we can treat the prism as a time delay beam former array whereby the delay is given by the variable length as the light passes through the glass. Hence one can view a prism simple as a tapped delay line at optical carrier frequencies![2] This is of some interest, and has been reported implicitly in [1], but a more complex example may be worth considering.

To this end in section 2 we will discuss Fresnel lenses, and show that they are a mixture of coherent and incoherent beam forming. In section 3 we draw similarities to subspace based energy detection [2]. We will also show that one can view grating lobes arising from phase only beam forming as similar to the diffraction pattern one obtains from the wavelength-modulus lens "scraping" that lies at the core of Fresnel design.

2 FRESNEL LENSES AS BEAMFORMERS

The Fresnel lens was invented in 1822, by the French physicist Joseph Fresnel [3]. Fresnel made the fundamental observation that at a fixed wavelength the gain of a lens (at broadside) is unaffected if the steering vector phase is limited to the principal range $[-\pi,\pi]$. Thus if we replace a rounded lens by scraping off glass every time the phase viewed by a signal impinging at broadside extends beyond the principal range we still get full gain. The advantage is that the lens is now must flatter and lighter. The disadvantage is that the lens is very hard to make! In fact with rare exception [5]. Fresnel lenses do not create full gain but rather revert to a constant phase stepwise approximation. Also, note that linear-phase only shifting is not sufficient in the near field. Before discussing the commonality of lenses and subspace beam-formers we will consider Fresnel lenses in isolation.

It is definitional that focusing within the near field requires more than a wave of linear phase shifts [3]. In this regime then, unless a large number of phase elements are used, a fair amount of phase tilt (quadratic phase) will fall across elements towards the edges, and unless tilts can be applied, the average phase error will be large, and performance will be poor. (Very near field requires amplitude taper as well, but this onset is rare in practice.)

As an example, let's consider a one dimensional case where 2 waves of curvature are used across the whole aperture which is subdivided into 9 equal sub apertures, or Fresnel "teeth". First to be addressed is the case of piston only phase control (corresponding to stair-wise as opposed to linear saw-tooth Fresnel sub-apertures). Figure 3 shows the desired quadratic vs. the realized phase profile

In Fig. 4 the piston-phase-shift-only realization of the desired wave front is shown to produce a spot (beam) pattern that is noticeablely weaker, slightly broader, and with distinct side lobes. The peak has fallen to about 65% of its original value, and the intensity would be down to the square of this or

1 *This work was the outgrowth of technical conversations and does not reflect findings of a formal funded program.*

2 *The dielectric variation complicates this for white light.*

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about 42%. This is as expected, since the peak intensity, or Strehl Ratio, is know to fall as exp($-\varepsilon^2$), where ε^2 is the mean square phase error, at least for small phase errors [3]. In this case the mean square phase error was 0.89 radians, or 0.23 wave and the predicted Strehl Ration is then 41%. Purely random phase aligment would drop the peak down to 33%, so we see that the piston approximation straddles perfection and chaos.

In the case where tilts as well as pistons can be applied to each sub aperture the resulting phase pattern can be piecewise linear. Not surprisingly, the phase errors are greatly reduced and the focused spot is greatly improved, virtually matching the black line in figure 4. One can show that the (small) phase errors are identical in each sub aperture in this case, whereas in the constant phase segment case the errors grow worse away from the center of the aperture. This really should be no surprise, since the example is one of constant curvature, and this constant curvature is the only part of the phase which a piecewise linear model cannot match within a sub aperture. Generally capturing the tilt suffices and higher order phase terms can be discarded on each Fresnel sub-aperture.

2-D FRESNEL PATTERNS

Consider a Fresnel lens as a sequence of concentric circles of prisms. Each ring puts the correct tilt on a ring of light to point it at the focus. This view ignores the tiny curvature presumably across each groove, but we will soon see this doesn't matter much for the example to considered. If the rings were to be spaced so each ring has a modulo-2π phase relationship with every other one, this lens would put the correct phase and tilt on each sub-aperture beamlet. Fresnel lenses commonly encountered on projectors and as magnifiers don't even attempt to get the modulo-2π phase relationships correct. This is in part because it would be difficult, but it is also because such an approach wouldn't work in (wavelength variant) white light. We will call such a lens an asynchronous Fresnel lens. So now we see a third way to look at phases and tilts as discussed above. What good does it do to put the correct tilts on the wave front, if the relative phases are essentially scrambled? Each ring of such a Fresnel lens works independently from all the others, and the wavelength of light is so short that this provides adequate performance for many applications.

Consider a typical Fresnel lens as used in an overhead projector as a concentrator. It's about a foot on a side with a one foot focal length and 2 grooves per millimeter. Then, for example, in a 2f to 2f imaging system where the magnification is unity, the diffraction limit will make the effective resolution about R* λ /d = 0.6m*0.5µm/0.5mm = 0.6 mm. Looking through such a lens, it is apparent that other distortions are far more important than diffraction in degrading the image quality. Across a 1 foot lens, there would be 600 grooves so spaced, and thus the lens would perform about 600 times worse than a diffraction limited lens (coherent vice incoherent gain). Effects due to the presence or absence of curvature, or focusing power, on each individual groove are also of this magnitude, since no focusing never adds more than the source diameter to a beam's width, and perfect focusing brings the spot size to the diffraction limit. Despite all this, for an optical wavelength of about 0.5 µm, 1000 waves fit across a 0.5 mm groove width, and the diffraction limit in angle space is thus 10 times finer than it is for the 1 foot W band antenna discussed in figure 2.

A "perfect" Fresnel lens which did match phases from groove to groove modulo-2π would be a challenge to construct, but modern MEMS manufacturing appears to be poised to enable this, at least in principal [4]. From the center to the edge of a 1 foot lens, there is a 1.5" difference in path length to a focus 1 foot away. This amounts to 70,000 waves of difference, and the perfect lens would need this many zones. The first zone would be more than 1 mm in diameter, and the last groove would be barely 1µm wide. The fractional optical bandwidth of such a lens is about the inverse of the number of zones, so such a lens would work well across less than 0.1 Angstrom of optical bandwidth.

It is hopeless to discuss phase error for the "imperfect" or common Fresnel lens. The average phase error would be on the order of magnitude of the number of zones of the "perfect" lens. With 70,000 waves of error, the performance of each ring is easily seen to be better than what might guessed for such a huge error, randomly distributed. In an asynchronous Fresnel lens, each groove has the right tilt and focus, but the phase jumps between rings is uncontrolled. There is a duality here, as the "imperfect" Fresnel lens phase error profile is conversely the phase profile of Fig. 3. This duality carries over to the Fourier domain, where the far-field of the "imperfect" Fresnel lens looks like the focused spot of the constant phase segment approximation to a quadratic phase profile.

It is interesting to reflect that in optics aperture is generally not a limiting factor, rather cost and lens precision is. This may explain why, unlike in radar, few optical designers "care" about reduced gain due to some incoherence.

NEAR FIELD FOCUSING AND FAR-FIELD PHASE

As a wave front is focused more strongly, the pattern at focus becomes smaller and smaller, but it doesn't change in shape or phase. For the case of a one dimensional square aperture, this pattern is shown in Fig. 4. However, the shape and phase of the far-field pattern does change. Figs. 5-6 show how the amplitude and phase in the far field evolve as the focus is drawn deeper into the near field. Amplitude is show as height, while the phase at each point is shown as color as given by the color wheel inset. As the pattern is focused more tightly into the near-field, the far-field phase is seen to unwrap some as the pattern spreads out.

3. BEAMFORMER AND SUBSPACE DETECTION

RF analogies come in two flavors:

(I) phase shift beamforming. Note that the Fresnel lens by nature has to be tailored to a particular wavelength. Thus all the artifacts of phase shift approximations arises as in radar, ambiguity lobes, gain loss off wavelength etc.

(II) Incoherently combined sub apertures. Note that in each piston or tilt the Fresnel lens is coherent. As we combine these sub apertures in an asynchronous Fresnel lens we loose coherency gain. This is akin to subspace processing, whereby one merely computes (whitened) energy in the absence of subspace steering vector [2] as we shall now see. This relation is best made more precise through equations. Let us take a slice through a circularly symmetric Fresnel lens made of linear tilt rings. Assume each ring has a width of N wavelengths. We can then express the pattern induced by the ring, as a function of azimuth, and wavelength, as:

$$b(\varphi) = \exp(j\pi\varphi N)\sin(\varphi N)/\sin(\varphi), \varphi = \pi\sin(\theta)/2 \qquad (1)$$

Where we have chosen coordinates so that the beam points at the azimuthal origin. Now suppose we have a perfect phasing from ring to ring. Then the composite pattern in the slice is given by a summation of the M beams (we assume far field focus):

$$b_\Sigma(\varphi) = \sum_{i=0}^{M-1} \eta_i(\varphi)\sin(\varphi N)/\sin(\varphi), \qquad (2)$$

$$\eta_i(\varphi) = \exp(j\varphi_i N)$$

Where $\varphi_i = \varphi i$ for a perfectly phase coherent synchronous Fresnel lens. In that case it is clear that the array gain in energy is (NM)^2. Now let us explore what happens when the phase is not perfect. Let's take the worst case scenario, where the phase is uniform, that is $\varphi_i =_d uniform[-\pi,\pi]$ and the d subscript indicates equality in distribution. Hence:

$$E[|b_\Sigma(\varphi)|^2] = E[\sum_{i=0,k=0}^{M-1} \exp(j(\varphi_i - \varphi_k)N)\sin^2(\varphi N)/\sin^2(\varphi)]$$

$$= E[\sum_{i=0}^{M-1} \exp(jN)\sin^2(\varphi N)/\sin^2(\varphi)] + cross\ terms$$

$$= M\sin^2(\varphi N)/\sin^2(\varphi) \qquad (3)$$

In contrast to (2) we see that the energy growth has been reduced by M which is the classic incoherent vs coherent gain comparison. Note that equation (2) has the form of a beam space combiner whereby each of the sinc function beams is combined using a beam combiner $\eta_i(\varphi)$ [2]. A well known technique in detection with unknown signal parameters is to compute the energy after projecting into a signal subspace [2]. Let v be the MN dimensional column vector which describes the steering vector of a wave front across a Fresnel lens comprised of M rings, each ring being N half wavelengths wide. [Whether the rings are contiguous or not is irrelevant in our discussion]. Let β be the Kronecker of sinc functions reflecting the lens kernel. Then we can express the output of the lens at a focal point using Kronecker products as

$$\vec{v} = \vec{a} \otimes \vec{b}, \vec{\beta} = \vec{c} \otimes \vec{d}, y = \vec{v}^H \vec{\beta} = (\vec{a} \otimes \vec{b})^H (\vec{c} \otimes \vec{d}) =$$

$$(\vec{a}^H \vec{c})(\vec{b}^H \vec{d}) = \vec{a}^H \vec{c}\sin(N\varphi)/\sin(\varphi),$$

$$\vec{a}^H \vec{c} = \sin(MN\varphi)/\sin(N\varphi)\ coherent, \vec{a}^H \vec{c} =_d N(0, M)$$

The coherent case corresponds to a single dimensional subspace (matched filter). The coherent case involves a random phase mixture, which by the linear distribution invariance of the normal distribution (which derives from the central limit theorem) is zero mean normal with variance M. We then immediately revert to (3).Thus we see then that subspace and asynchronous beamformers behave identically! The grating lobe aspect of Fresnel lenses is straightforward to develop by extending (2)-(3) to the non-uniformly spaced case, i.e. with wavenumber mismatch.

The perfect Fresnel lens is rarely contemplated since it is difficult to design. Interestingly optics is a very old science, with much lens crafting based on heuristics. One dares to imagine that optics would be viewed differently (conceptually if not physically) if only it had waited until recently to be developed.

4 CONCLUSIONS AND IMPLICATIONS

We have discussed Fresnel lenses and subspace beamformers, and their common theoretical linkages. One of the more intriguing observations is that a common Fresnel lens, such as one finds on an overhead projector, is an incoherent subspace beamformer. The incoherence arises from two sources, the white noise fractional bandwidth which disallows the phase approximation to true time delay, and the design tolerance of etching glass. The former is very familiar to radar engineers. Since customers are willing to pay much for radars the design tolerance is much tighter so the etching constraints have less cache! Further the higher wavelengths in radar make for much less demanding manufacturing.

Another observation of viewing lenses and antennas in a common framework is that we see that Fresnel lenses, by virtue of the modulus truncation, will suffer grating lobes off axis. This is inherent in the Fresnel design, unlike other manufacturing limitations discussed above. This may help to explain why Fresnel lenses have tended not to be advanced for remote imaging applications [5].

The convergence of optical lens and RF beamformers will only increase. This is due to two technical evolutions, some might say revolutions.

First, high frequency radars are emerging that have substantial electrical apertures. As an example the Green Banks telescope (though passive) has, at W band, roughly 30,000 wavelengths across the aperture. A near IR system, operating at 1 micron, with a 10cm lens, would have 100,000 wavelengths. Thus wideband and near field effects will become more prominent in radar. This will lead to opportunities to scavenge design concepts from optics for radar use.

Second, the optics world is changing with the rapid pace of nanotechnology. This technology is increasingly allowing very precise machining. As such the option will be quickly availing itself of allowing for coherent Fresnel optical systems. Since radars tend to be coherent, This will lead to opportunities to scavenge design concepts from optics for radar use.

Convergence of optical and RF beam forming increases competition associated with a merging of disparate fields. Having a larger talent pool upon which to draw for insights is salutary, whether in capitalism or science.

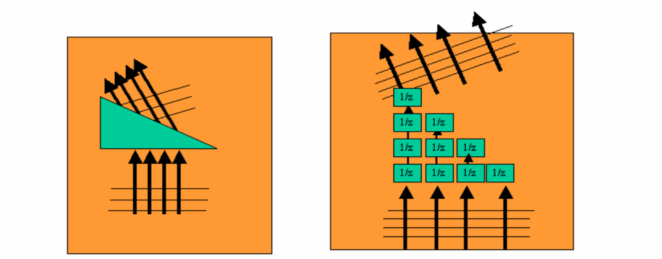

Figure 1 (a) a prism (b) beamformer
The light on the left hand side of the prism has to travel longer than the light on the right. This “swings” the light to the left. On the beam-former shift registers replace lattices.

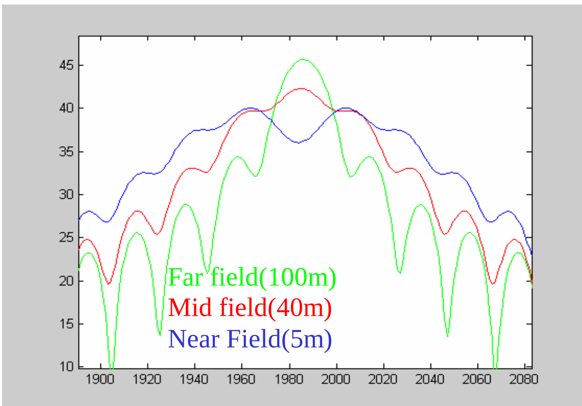

Figure 2: Antenna pattern for a near field true time delay beamformer. The antenna is W band, 1 foot in diameter. In all cases the plot (vertical in dB power) is the respond to a far field source from a bearing, ranging in this zoomed image across 5% of physical space.

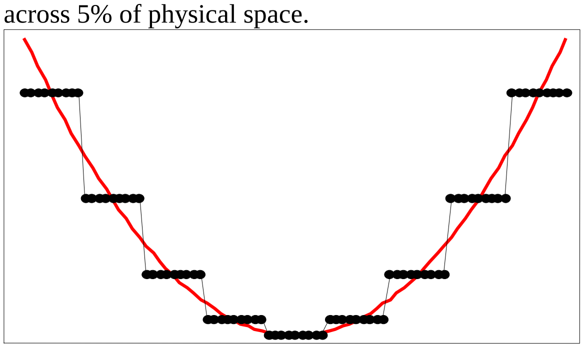

Figure 3. A desired phase profile vs. optimal constant phase segment approximation. In most Fresnel lenses the individual lens segments are uncurved, resulting in a linear phase. Phase tilting is viable approximation to the true desired quadratic curvature.

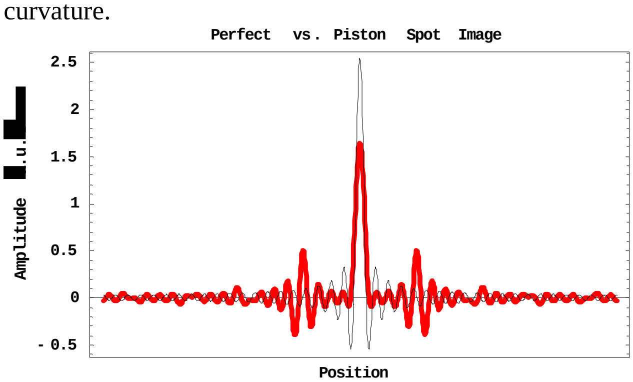

Figure 4 (previous column). The resulting focused spots for the phase profiles shown in Fig. 3. The roughly 50% degradation is well on the way to the incoherent, random, phase combining across beamlets, as per eq. (3)..

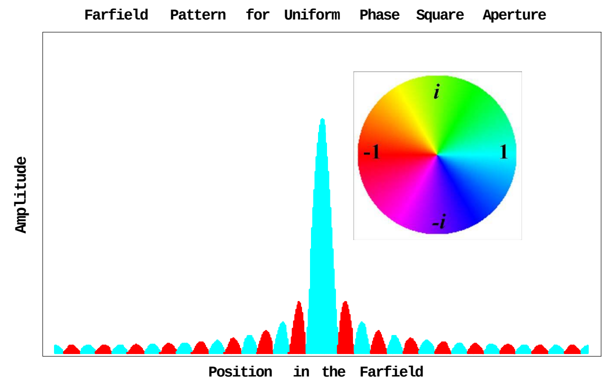

Figure 5. The sinc amplitude pattern of a one-dimensional square aperture with uniform phase. (far field beamformer). Note the phase reversal from lobe to lobe.

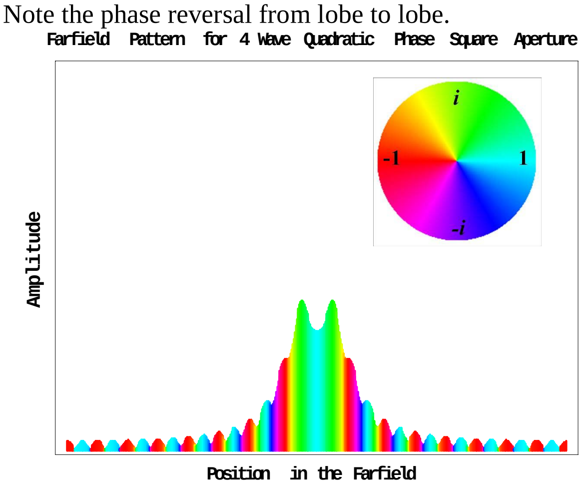

Figure 6. The amplitude pattern of a one-dimensional square aperture with 4 waves of curvature across the aperture. The bimodality explains the “ghosting” one sees from blurry imagery and near-field clutter. Note also the nuanced phase alterations. This non-uniform phase fluctuation is what disallows a simple tilt sub-arraying to be lossless, as well as disallowing a Fresnel lens to be focused at variant ranges. From that point of view an asynchronous design enjoys a certain robustness.

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